Decoding the Graph-Equation Correspondence

a first example

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In mathematics, starting with basic Algebra, the connection between an equation and its graph plays an important role. The visual representation of a function or equation as a graph offers many advantages including giving, at a glance, a "feel" for what is going on in the problem where the graph is relevant. For example, if the graph represents the cost of producing a certain number of objects assuming a certain fixed number are sold, a glance at the graph will indicate estimates for optimal production and will immediately communicate, for example, that once the production is high enough, producing more costs more. The shape of the curve is essential. There is a correspondence between the equation that produces the graph and the graph, in the sense that certain features of the graph can read off immediately from the equation, if it is in an appropriate form. In mathematics leading to engineering type applications, this connection is essential.

Step 1. What is the bottleneck to learning in this class?

The bottleneck is a combination of understanding the equation/graph relationship and in particular, recognizing "interesting" features of graphs (here we will focus on the parabola) as well as the identification of these features in the corresponding equation. Students seem to struggle with noting interesting features due to lack of experience and examples and with the graph/equation connection due to the abstract nature of the equation as well as possible fear of variables. The idea here is that if a student can master the parabola (in its various representations) and their "dictionaries", then this will serve as a solid foundation for all other instances of the equation-graph connection. Conceptual problems that students have in their mathematics courses, including the all-important derivative in Calculus, has its origins in algebra and not understanding or being comfortable with the equation-graph connection. Further, smooth functions, often look like parabolas near "interesting" points of the graph. This bottleneck includes recognizing "interesting" features of a graph, and recognizing the corresponding feature of the equation (which may not be obvious), that is to say understanding the algebra-geometry, or equation-graph, dictionary in the case of the parabola for example recognizing the importance of the vertex in graphing a parabola and to be able to find the vertex by using an equation alone.

Step 2. How does the expert do these things?

This largely depends on the problem at hand. But let's say the particular question to master is to graph a parabola $y = ax^2 + bx + c$, for various fixed a, b, and c. This is recognized by the expert as a parabola due to the form of the equation (it is quadratic in one variable and linear in the other). An image of the basic parabola $y = x^2$ comes to mind. The features of the basic parabola are the symmetry, the vertex (the lowest or highest point), the "width", and whether it "opens upward" or "opens downward". Many parabolas may also have places where it crosses the x- axis, which may be of special interest in applications (where a ball that is thrown hits the ground, for example).

The thing that is quite clear in the expert's mind is that the graph is ideally a "picture" of all of solutions (in reality it never represents all of the solutions, but they are implied). This is the most fundamental issue for students to understand. The understanding comes from a discussion and associated practice of how to recognize a solution and how to place it on the graph (the coordinate plane can be modeled by a map of midtown Manhattan, with the "dictionary" of descriptions of locations vs coordinates), and a re-emphasis of this connection whenever possible.

The expert keeps in mind the goal of graphing a parabola and identifies pieces of information that would be helpful, then decides which are the easiest pieces of information to get that will result in a satisfactory graph. This requires experience and anticipation of what information would be sufficient, and how one goes about getting that information. The expert knows how to get every piece of possible information through transforming the equation by factoring or completing the square and recognizing which features can be gleaned from each form.

The students can gain experience recognizing features of various types of graphs by giving many examples and discussing why two graphs are similar or dissimilar.

The order of the features depends on the problem, much like a detective solving a mystery. Information is gathered and then put together as soon as it is enough to form a graph. A reminder of a similar process with lines can be given (features being the slope, and x and y intercepts, or a specific point that the line may go through).

We look at the easiest thing to recognize first: if a is positive then it opens upward, otherwise downward. Give the example of $y = x^2$ and $y = -x^2$ (by plotting points) to have students discover that while the first graph opens upward the second opens downward.

The second feature is the vertex. This can be obtained in a number of ways: Either first find the zeros (solve $0 = ax^2 + bx + c$ in the easiest possible way) and use the symmetry to find the $x-$coordinate of the vertex. Then use the equation to find the $y-$coordinate. Or we complete the square, by writing $y = ax^2 + bx + c = a(x - d)^2 + e$ and noting that the vertex is (d, e). This relies on understanding the basic transformations of graphs and their corresponding equations.

The third feature is the symmetry. This is the vertical line passing through the vertex.

The fourth is the width: The basic shape can either be recognized through the transformation ideas above or by plotting a few points left and right of the vertex–noting the symmetry. Reinforcement of the importance of the sign of a should be given at this point.

The fifth is the location of the zeros (especially if applications motivate the problem): These are found by solving (solve $0 = ax^2 + bx + c$ in the easiest possible way)–this may have been done in finding the vertex. If there are zeros, they should be identified on the graph, and the information should be consistent with the other steps. (Some information obtained will be redundant just like a detective may find information that is consistent with prior information but should not be inconsistent). The connection with the vertex and symmetry should be re-examined after this step.

In the expert's mind, these things may present themselves seemingly simultaneously as in a flash. And with a particular problem, different features might present themselves as more important. This comes with experience and great familiarity, presumably, like basic mechanics of throwing a ball is not really considered by a professional ball player (unless issues of some sort arise).

Step 3. How can these tasks be explicitly modeled?

The first thing with the parabola is to demonstrate it's general shape and features as well as the value of graphing the shape on a coordinate plane by having two students toss a bean bag to each other and have the other students draw a picture of its path. Discuss and agree on a picture drawn on the blackboard. In order to discuss relative location as well as the fact that certain transformations change features of the graph yet the transformed shape is still a parabola, ask the volunteers to take two steps to the left and repeat. Does the picture look the same? What about two paces to the right? Introduce a coordinate plane on the picture on the blackboard so that specific locations can be discussed. Students, I expect, will agree (after discussion) that this aids in answering questions (how high did the ball go), where would it have hit the ground if the student hadn't caught it? Discuss the idea of a map, say of midtown Manhattan as an example of a coordinate system and how actual locations are described by coordinates. Give an example of a simple equation in words describing a person's possible location (x th street and $x-2$ nd avenue ($y = x - 2$) where y represents avenues and x represents streets) and draw possible locations of that person, illustrating the notion of solution and the graph.

Next the basic equation $y = x^2$ should be discussed in detail (with its features as described in the previous section). And the effects of adding or subtracting a number to x or y (or multiplying the square term by a number). This can be accomplished by experimenting with DESMOS or by graphing by hand as time permits. DESMOS is a freely available app that most if not all students will have access to. It is easy to use and provides a useful tool for experimentation.

Then numerous examples of $y-e = a(x-d)^2$ should be discussed and features discovered by students in groups and individually. Zeros should be found in particular cases in various ways (which should be a review for the students). Symmetry, the vertex, and the point in between the zeros should be discovered through questioning (in groups and individually). Tables to help with the shape should be made and plotted on the graph. The inverse problem of "given a graph, write the equation" should be given as a challenge in groups. In fact, there is a problem in WeBWorK that engages the students in this inverse problem. While in this discussion, this appears as an option for students, it is actually very important in solidifying this graph/equation connection, along with understanding what "interesting" features.

Finally, the case of $y = ax^2+bx+c$ should be given (with various numbers). Features and how to go about finding them should be discussed. If completing the square isn't suggested by the students it should be suggested by the instructor and practiced as a method (since it will be used with other conic sections). Several applications should be given. Questions asked that pinpoint various features can be discussed. This is modeled by giving several examples with increasing student contributions to the solution.

Step 4. How will students practice these skills and get feedback?

Students will practice through class discussion, group assignments (discovery questions), and individual assignments, including homework which may or may not be web based. They will practice increasingly difficult combinations of ideas in each of these contexts.

Similar steps should be applied to other shapes besides the parabola, as they come up. The connection to the parabola should be pointed out (by questioning) in each case.

The feedback will be in discussion form (further questioning to allow students to correct misconceptions), or by group members giving each other feedback, by written feedback by the instructor, electronic immediate feedback if the homework is online, or class feedback (as in Austin's Butterfly video `https://www.youtube.com/watch?v=dOSiU42P8Gc`) for students who present solutions to the class.

Step 5. What will motivate the students?

I would share the videos of Austin's Butterfly video `https://www.youtube.com/watch?v=dOSiU42P8Gc` and the growth mindset videos `https://www.youtube.com/watch?v=J-swZaKN2Ic` and `https://www.youtube.com/watch?v=Yl9TVbAal5s` to encourage students to think about their own success and thinking process. I may show one of the latter two in class and have the students watch the others a week or so later as homework (so that students who registered late will also benefit early). Depending on student response I may show one when they seem to struggle with motivation. I will also have a discussion concerning the explicit versus tacit knowledge and the importance of good practice.

For mathematics-specific motivation, I may have a discussion on the importance of this example in all future mathematics and in applications as well as in reading common articles (understanding graphs and their interpretations– students can be asked to bring in examples of graphs in common newspapers). There will also be a discussion concerning expectations of the difficulty of mathematical problems in order to avoid student frustration. It is the case that many students in these courses are used to one step questions. They will be told that there are many steps and it is a little like detective work. They should take each problem as a challenge and learn to ask questions to themselves (like a detective might). Like using a map for directions, they must keep their beginning and their end in mind simultaneously. They all can be successful if they are patient and persistent. Focusing on the concepts is critical for success and while focusing on the concept, rather than the answer or procedure, may seem more difficult at first (with less payoff) the end benefit is great and will last throughout their mathematical career.

Step 6. How well are the students mastering these learning tasks?

Dividing the bottleneck into pieces as in step 3, will allow each step to be assessed. Group discussion can be monitored for understanding of each step as well as how well the students answer questions posed to the class, and individual work should be checked). Only after trying this particular process will this question be answered and sub-bottlenecks discovered and dealt with. The hope, though, is to repeat this process with all equations/curves in the course so that the equation/graph relationship is comfortable and that the students begin by looking at "important features"

Step 7. How has this approach to teaching parabolas changed as a result of this process?

The praise I give will be deliberately focused on a growth mindset. The class will more often have discussion (though perhaps brief) on the learning process and attitudes. The time and effort spent on understanding the basics (non-technical issues) will likely be much greater and will include modeling when possible. I expect to give more conceptual questions to accomplish mastery of these less technical aspects of the problem. The students will experiment with DESMOS (perhaps as in-class group work).

Appendix

A. Expert thinking

Here we look at the bottleneck in the context of graphing a parabola. A student will need to know what it means to have a solution and be able to recognize the relationship between an equation and the geometric representation of solutions. They will need to be able to recognize the important features of a graph (the vertex, the symmetry, the y-intercept and the x-intercepts (roots) and the general shape of the solution set) and be able to see the manifestations in the equation. This may involve computations like completing the square, factoring, or using the quadratic formula.

Expert break down of bottleneck: First look at the problem and try to see which of the ways is best to proceed. Find the vertex by either averaging roots or completing the square. Note the sign of the leading term. Have a rough graph in mind. Note the shifting of the parabola compared to basic parabola. Find the intercepts by appropriate method (completing the square or factoring). Graph the information (perhaps with the help of specific points just left and right of the vertex). See modeling and rubric in the appendix. Breaking down examples in order for students to graph various parabolas on their own (use modeling for appropriate parts as described in assignment 3). The parenthetical letters are referring to the highest level and usually includes all lower levels as well. The task is to graph a general quadratic function. Here the letters in parentheses refer to Blooms taxonomy (Synthesize, Evaluate, Analyze, Apply, Comprehend, and Remember) and in the cases below the indication also includes lower levels on the taxonomy (for example all of the below include Remembering).

1. (C) What are solutions? Go back to lines, and give an example of parabolas. Demonstrate (as on a map) of the relationship between solutions and graphs. Assessment is done by asking students questions to make sure understanding is achieved by all.

2. (C) Purpose and need: Give examples of graphs used around them... sound waves, planetary motion, modeling... locally the graphs look curvy—like parabolas. Quantities changing in time are often graphed. Equations are a compact formulation of a set of solutions, where as a graph gives a feel for solutions. Questions asked during discussion.

3. (An) Features of graphs, in general, and specific to the parabola. Max, min, intercepts, increasing, decreasing, convexity, symmetry. Plotting points as a method. How do we choose points, and how many points are enough... discussion. Peg board and yarn example.. Students volunteer answers to questions.

4. (An) Focus on the basic parabola and its features. Plot solutions. (Have students work examples)

5. (S) Locate vertex, symmetry, intercepts, opening direction on graph examples. Have students volunteer answers.

6. (S) Locate vertex, symmetry, intercepts on equations in form $y - c = a(x - b)^2$. Graph examples. Model translation process by ball throwing changing positions, and relative positions noting the shape remains similar. Have students volunteer answers.

7. (E) Completing the square to put a general parabola into the form $y - c = a(x - b)^2$. Practice increasingly difficult examples with and without help from me. Have students present work and be assessed by other students for accuracy and presentation.

8. (E) Consider finding roots by factoring and also by solving the quadratic formula. Have students also recognize symmetry axis based on roots (if roots exist). Practice and have students present and other students comment.

9. (Ap) Fine tune shape by plotting points left and right of parabola. Have student discover which points are appropriate to assist with the shape.

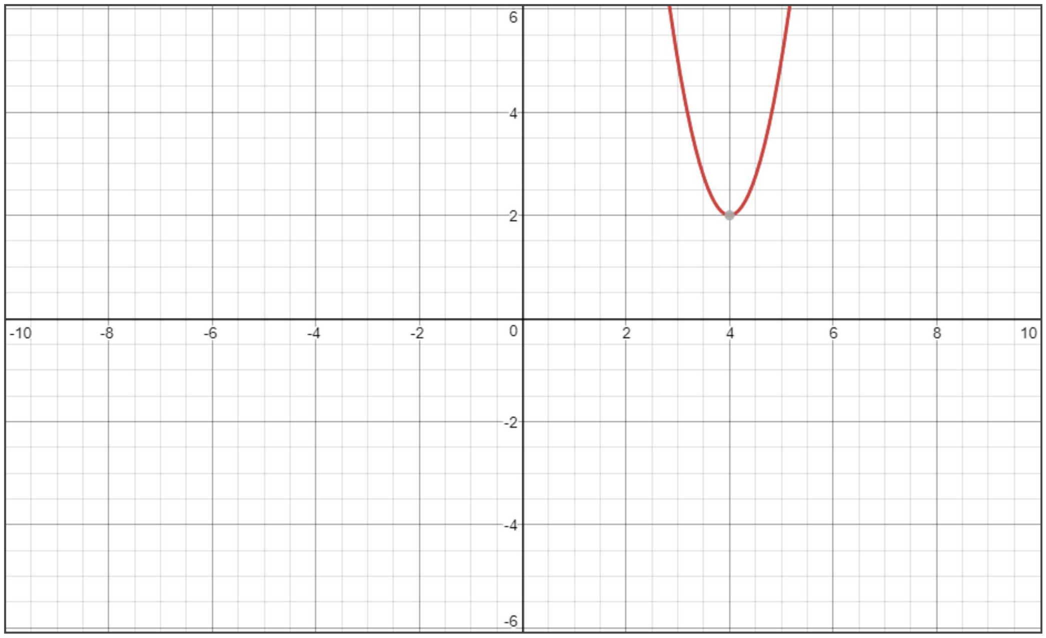
10. . (E) Discussion: what if there are no real roots? Discussion and conclusions from students.

11. (S) Putting it all together: Graph parabolas and (E) write equation for a graph that is given. Look at student work.

12. (S) Give applications with questions involving interpretation of zeros and vertex. Look at student work.

13. (E) Discussion on different components and advantages or necessity of certain steps to create and interpret a graph.

B. DESMOS example



1 $y-2=3(x-4)^2$

C. Rubric

The bottleneck is understanding the graphing/equation connection and the recognizing "interesting" features. In the first nonlinear instance of this occurs with parabolas. This topic is one that many students struggle with for various reasons. It is the first nonlinear example of graphing the solution to an equation. This relationship between the numerical solutions to an equation and its graph is essential in mathematics and serves as a source of difficulty in courses up through calculus. It can involve solving equations, understanding shifts of equations, as well as understanding representations of the solutions on the coordinate plane. This topic is important because it includes many foundational skills that are important in both high school and college mathematics courses including but not limited to:

1. Understanding how an equal sign in an equation with 2 variables establishes the relationship between the 2 variables
2. Substituting values into an equation and evaluating
3. Creating a table of values from an equation
4. Plotting points and graphing lines from plotted points
5. Finding x and y intercepts
6. Factoring
7. Completing the perfect square
8. Shifting, expanding and compressing graphs
9. The relationship between the shifts and scalings of graphs and their corresponding equations.
10. Finding the maximum or minimum point of a parabola using and equation
11. Identifying the characteristics of a parabola.

When faced with a problem of graphing an equation that is quadratic in x, there are several possible approaches. One uses a table... filling out a table of solutions can require knowledge of the x coordinate of the vertex. This can be found either by completing the square (and recognizing it as a shift of a standard parabola) or finding the roots and averaging. Intercepts should be found and plotted (this will involve either the quadratic formula or factoring). Additional solutions may be found near the vertex by finding y given various values for x. These should be plotted on an x and y axis and a curve should be drawn and labeled. It should be checked that the parabola opens in the correct direction.

The development of a Decoding Rubric begins by making a list of the errors that students have made attempting to complete a particular assignment that is an example of the bottleneck you are working with. To create this list, the instructor may examine student work on actual assignments, or the instructor can create the first rubric using errors s/he expects students to make adding to the rubric as s/he goes along. The errors, in effect, are the bottlenecks or what students do not know to do.

Competencies	Unacceptable	Developing	Proficient
Locating the vertex	Not properly identifying the x coordinate of the vertex	Recognition that completing the square will lead to the solution, or finding the roots will be a step toward finding the vertex	Either complete the square properly, or average the roots.
Plotting on the coordinate plane	Unable to plot appropriate points	The x and the y coordinates are swapped, or some points are correct.	Locate the vertex and plot "convenient" points near the vertex, or use an appropriate shift
Recognition of the solution as a graph	Unable to recognize solutions as points on the graph	Some solutions are plotted but a curve is not drawn	Remind oneself of the connection between solutions and the graph of solutions
Root finding	Unable to located roots	the numerical roots are found but not identified or misidentified on the graph	Identify roots by noting that $y = 0$ for such solutions and either factor or solve the quadratic equation
Recognition of role of leading coefficient	Parabola opens the wrong direction	NA	Check the sign of the leading coefficient

Plotting points equidistant to the vertex	All points plotted on one side of the vertex and not near the vertex	Only one point plotted on each side of the vertex or point plotted at significantly different distances from the vertex on each side resulting in an asymmetrical sketch	Identify the vertex, pick 2 or three x values to the let as and right of the vertex which are at the same distance from the vertex then plot points for each value picked
Using appropriate lines to sketch	straight lines used to connect all the points on the sketch	appropriate curves were used to connect some but not all points on the sketch	curved lines used when appropriate to plot points and create the general shape of the parabola
Improperly labeling axis	the x and y axes were switched	the increments were irregular	each axis was properly labeled and increments of each axis were chosen to allow the vertex to fit in the graph.

D. Modeling Examples

1. Locating the Vertex: Show an image of Stephen Curry and or Lebron James shooting  

Explain to the class how shooting coaches always recommend that shooters release the ball at the highest point of their jump. If we graphed the shooters height overtime the graph would look like a parabola with a negative leading coefficient. This peak of the jump is called the vertex of the parabola. If given a parabola which open up the lowest point is our vertex.

Method A. Assuming students have learned the shifts or graphs and how they impact the basic parabola $y = X^2$ then I would proceed as follows:

* Project a 1- 3 graphs of parabolas with equations in vertex form from desmos.com (https://www.desmos.com/calculator/w8erhdmezz)

* Have students identify the vertex verbally and describe what elements of the equation are similar.

* Review 2 - 3 examples of identifying the vertex from equations which are already in vertex form and check with desmos.com graphing tool.

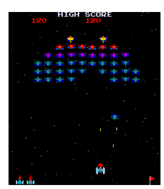
* Briefly review examples of perfect squares

* Demonstrate the process of developing a perfect square using $(\frac{b}{2})^2$

*Demonstrate completing the perfect square to create an equation which is in vertex form and identifying the vertex.

Method B. After students have mastered method A, I could provide them with a shortcut / check in which they could determine the x- coordinate of the vertex by using x= -b / 2a and find the y – coordinate of the vertex by substituting the x- coordinate into the equation. I would demonstrate this method using some of the same examples I did using Method A so scholars could see how they match up.

2. Plotting on the coordinate plane:

Show a short video or image of Space Invaders Gallactica (1979). 

Explain to students how in the game you had to move the space ship back and forth from left to right then shoot up at the monster or space invader you wanted to hit. If you didn't shoot them in time they would come down towards you.

Pause the video or look at the image and ask a student what would they do if they had to shoot a certain monster. After the student describes their process ask 2 more students he same question. Explain to scholars that in each situation a student described moving left or right 1st then shooting up at the monster. This relates to plotting points because your x coordinate which is the 1st number in a pair of coordinates always determines whether you go left or right from the origin. Positive + = Right and Negative - = Left. After you move left or right in the game you must either shoot up or the monsters will come down to get you. Shooting up = positive points so Positive + = Up and monsters coming down is negative so negative = Down. In short when plotting points go left or right 1st just like your ship, then go up or down based on the 2nd or y coordinate.

Demonstrate several examples of plotting points while referencing the Space Invaders Gallactica analogy.

3. Recognition of the solution as a graph: Give a similar problem in terms of a riddle: I am standing at xth avenue and yth street. Where could I be? Put solutions on a map (label avenues with negative numbers so that numbers increase from left to right) drawn on the board. Write this in an equation and put on a graph that overlays the map. Model the infinite nature of the solutions and the need for a graph by comparing a description of a particular apple (that will distinguish it from all other apples) and compare with a photograph: which is more likely to lead to the correct recognition of that apple in a crowd of apples. A picture is worth a thousand words,… rather,.. a picture is worth an infinite number of words.

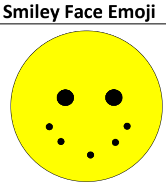
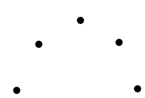
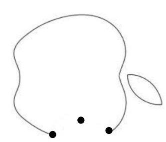
4. Finding roots: Present as an application of when a thrown object hits the ground. How do we find roots? Give examples of the zero product rule, completing the square, and the quadratic formula.

5. Leading coefficient: Look at large x- behavior to see if the parabola opens up or down. Importance: a rough sketch of the parabola (helps to check for errors)—just like estimating a the cost of a basket of groceries. This together with the vertex is enough to get the first rough idea of what the parabola will look like.

6. Plotting points equidistant to the vertex: Use a pegboard…perhaps with pins and a string. Put pins at solutions and a string round the pins. How many pins will be needed to get an idea of the shape and position of the graph? Two pins would be enough for a line, though three would help you draw. How many is enough for a circle (knowing it is a circle,.. usually 4 is enough to keep you on track—but not any 4 pins… 4 specific pins). How many are needed for a parabola? Too many takes time,.. Too few gives the wrong impression… Note the symmetry of the parabola and start by looking for points with x-coordinate one unit away from the x-coordinate of the vertex. Is that enough? No,.. it could look like a "V". Now look for points with x-coordinate two units away from the x-coordinate of the vertex. Is that enough? Probably… but another pair might be helpful. A pin at the vertex and two or three pins on each side of the vertex should be enough, knowing what a parabola looks like.

Where these specifically will be discussed in class may depend on questions the students have.

7. Using appropriate lines to sketch: Projects several images on the board with dots that need to be connected to complete the figure. These images should have titles above them and include dots which will form a parabola if the dots are connected correctly. Example below:

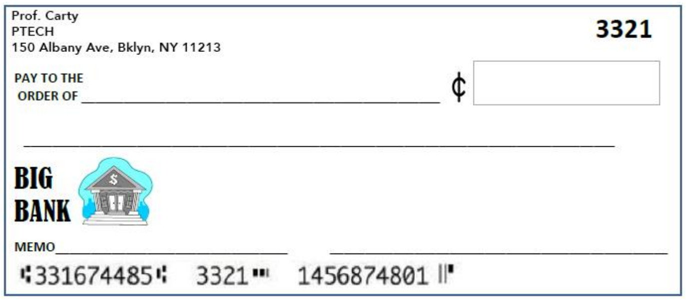
Connect the dots to properly complete each figure below			
Smiley Face Emoji	**Snow-board Half Pipe**	**Path of soccer-ball**	**Rotated Apple Logo**
			

Project a 2nd image of the same images as above with the dots connected with all straight line segments. Ask the students what they think about how you have drawn line segments to connect the dots. After hearing several reasons about how straight line segments are inappropriate for the images, state to the students that same practice of using curved lines to connect the dots for the figures also applies for graphing parabolas.

Project the 1st image again as seen above and connect the dots using curves. State to the students that when you a graphing a parabola you should see it as part of an emoji, half pipe, apple logo etc, etc and always use curves where appropriate. Use this moment to explain the parabolas go on to infinity on each end so you draw arrows on each end to indicate that.

Demonstrate several examples of completing the sketch of a parabola from plotted points while referencing the analogies above.

8. Labeling Axis: Inform students that the student who can give the best and clearest anology on how to label axis will receive a check for $100. Take down ideas from students on how they remember to label their axis correctly. It is very likely that students will provide may relevant anaologies which can be used to help their classmates. Once a winner is chosen state to the class that you are going to project a check and fill it out on the board for evidence then project the check image below and quickly fill out the students name, the 100 and sign it.



Then ask students if they witnessed the signing of the check. If students do not see it inform them that the check is actually for 100 cents and it was improperly labeled..

Based on the quality of student responses in the previous activity you can also provide them with the following ways to remember to properly label axis.

A. Make a connection between the Space Invaders Gallactica as used in item 2 of this document.

B. When you want to x something out that you have written you normally make a single horizointal line through it. The X- axis goes from left to right in the same way.

X ~~something~~ out = line from left to right = x –axis

Demonstrate crossing something out and write x – axis = left to right next to it.

Demonstrate labeling 2 or 3 sets of axis while refenrence one of the analogies submitted by students or the one abput crossing out.

E. Modified Syllabus

NEW YORK CITY COLLEGE OF TECHNOLOGY
The City University of New York

DEPARTMENT:	Mathematics
COURSE:	MAT 1275
TITLE:	College Algebra and Trigonometry
DESCRIPTION:	An intermediate and advanced algebra course. Topics include quadratic equations, systems of linear equations, exponential and logarithmic functions; topics from trigonometry, including identities, equations and solutions of triangles.
TEXT:	Custom Text by McGraw-Hill containing material from Intermediate Algebra, 3rd edition by Julie Miller, Molly O'Neill, and Nancy Hyde and Trigonometry, 2nd edition by John Coburn
CREDITS:	4
PREREQUISITES:	MAT 1175 OR for New Students, scores of at least 45 on the Pre-Algebra part and 45 on the Algebra part of the CUNY Assessment Test in Mathematics.

Prepared by:
Prof. H. Carley
Prof. P. Deraney
Prof. A. Douglas
Prof. M. Harrow
Prof. L. Zhou
Spring 2013

A. Testing/ Assessment Guidelines:
The following exams should be scheduled:
1. A one-hour exam at the end of the First Quarter.
2. A one session exam at the end of the Second Quarter.
3. A one-hour exam at the end of the Third Quarter.
4. A one session Final Examination.

B. A scientific calculator is required.

Course Intended Learning Outcomes/Assessment Methods

Learning Outcomes	Assessment Methods
1. Solve • Linear and fractional equations. • One-variable quadratic equations by factoring, completing the square, and the quadratic formula. • Radical and exponential equations. • Systems of equations.	Classroom activities and discussion, homework, exams.
2. Perform operations with and simplify polynomial, rational, radical, complex, exponential, and logarithmic expressions.	Classroom activities and discussion, homework, exams.
3. Apply their knowledge of algebra and trigonometry to solve verbal problems.	Classroom activities and discussion, homework, exams.
4. • Solve problems involving right and oblique triangles. • Prove trigonometric identities. • Solve trigonometric equations. • Graph the sine and cosine function.	Classroom activities and discussion, homework, exams.
5. Apply the distance and midpoint formulas and determine the graphs of circles and parabolas	Classroom activities and discussion, homework, exams.

General Education Learning Outcomes/Assessment Methods

Learning Outcomes	Assessment Methods
1. Understand and employ both quantitative and qualitative analysis to solve problems.	Classroom activities and discussion, homework, exams.
2. Employ scientific reasoning and logical thinking.	Classroom activities and discussion, homework, exams.
3. Communicate effectively using written and oral means.	Classroom activities and discussion, homework, exams.
4. Use creativity to solve problems.	Classroom activities and discussion, homework, exams.

Mathematics Department Policy on Lateness/Absence

A student may be absent during the semester without penalty for 10% of the class instructional sessions. Therefore,

If the class meets:	The allowable absence is:
1 time per week	2 absences per semester
2 times per week	3 absences per semester

Students who have been **excessively absent and failed the course at the end of the semester will receive either**

- the WU grade if they have attended the course at least once. This includes students who stop attending without officially withdrawing from the course.
- the WN grade if they have never attended the course.

In credit bearing courses, the WU and WN grades count as an F in the computation of the GPA. While WU and WN grades in non-credit developmental courses do not count in the GPA, the WU grade does count toward the limit of 2 attempts for a developmental course.

The official Mathematics Department policy is that two latenesses (this includes arriving late or leaving early) is equivalent to one absence.

Every withdrawal (official or unofficial) can affect a student's financial aid status, because withdrawal from a course will change the number of credits or equated credits that are counted toward financial aid.

New York City College of Technology Policy on Academic Integrity

Students and all others who work with information, ideas, texts, images, music, inventions, and other intellectual property owe their audience and sources accuracy and honesty in using, crediting, and citing sources. As a community of intellectual and professional workers, the College recognizes its responsibility for providing instruction in information literacy and academic integrity, offering models of good practice, and responding vigilantly and appropriately to infractions of academic integrity. Accordingly, academic dishonesty is prohibited in The City University of New York and at New York City College of Technology and is punishable by penalties, including failing grades, suspension, and expulsion. The complete text of the College policy on Academic Integrity may be found in the catalog.

MAT 1275 College Algebra and Trigonometry

Text: McGraw-Hill Custom Textbook containing material from Intermediate Algebra**,** 3rd ed., by Miller, O'Neill and Hyde (sessions 1-16 and 26-29) and Trigonometry, 2nd ed. by Coburn (sessions 18-25).

Session	Topic	Chapter, Section, and Pages	Homework
1	Properties of Integer Exponents Adding and Subtracting Rational Expressions	Chapter 4, Section 4.1, pages 314-318 Chapter 5, Section 5.3, pages 431-438	p. 321: 11-29(odd),33,35,41,47,63,67,75 p.439: 7-23, 27-49 odd
2	Complex Fractions	Chapter 5, Section 5.4, pages 441-446	p.447: 9-15,17-23 odd, 31,33
3	Fractional Equations	Chapter 5, Section 5.5 pages 449-455	p.445: 9- 33 odd
4	Roots and Radicals Rational Exponents	Chapter 6, Section 6.1, pages 492-498 Chapter 6, Section 6.2, pages 503-507	p. 500: 9-37 (odd),59,65,67,79 p. 508: 11,15,19,25,29,33,41, 45,53,65,73,81,93
5	Simplifying Radical Expressions Addition and Subtraction of Radicals	Chapter 6, Section 6.3, pages 510-514 Chapter 6, Section 6.4, pages 517-519	p. 515: 9,13,17,21,25,33,39,55,59,63,79 p. 520: 15,19,23,35,37,41,51,55,57,61,79
6	Multiplication of Radicals	Chapter 6, Section 6.5, pages 522-526	p. 528: 11,17,19,21,23,25,29,31,35,37,55,57,61,63, 67,77,79,87
7	Division of Radicals and Rationalization	Chapter 6, Section 6.6, pages 536-537 (skip examples 4 and 6)	p. 538: 11,13,17,21,31,35,39,53,57,63,67,71,77,81
8	Solving Radical Equations	Chapter 6, Section 6.7, pages 540-543	p. 547: 11-16,21-24,37-42
9	**Administer First Examination** Complex Numbers	Chapter 6, Section 6.8, pages 550-557	p. 558: 15-27,31-35,53-57,61-69,81-89 odd
10	Quadratic Equations The Square Root Property and Completing the Square	Chapter 4, Section 4.8 pages 388-390 (omit example 2) Chapter 7, Section 7.1, pages 574-579	p. 398: 17-36 all p. 580-581: 3-17,21-27,31-53 odd
11	The Quadratic Formula	Chapter 7, Section 7.2, pages 583-585,588-594 (Derive the quadratic formula)	p. 595: 9-25,39-55 odd, 69,73,77,81,85
12	Applications of Quadratic Equations	Chapter 4, Section 4.8, pages 392-394 Chapter 7, Section 7.2, page 586	p. 398: 61,65,67,69,71 p. 595: 41,43,47
13	Graphs of Quadratic Functions	Chapter 7, Section 7.4, pages 604-612 Chapter 7, Section 7.5, pages 618-621 Do all modeling here in an order that will depend on student questions. Suggested order is 3, 1, 2, 5 4, 6 (completing the square	p. 613: 11-15,19-23,29-35, 45,47,51-61 odd p. 624: 17-23 odd, 29,31,37,41,43

Session	Topic	Chapter, Section, and Pages	Homework
		and shifting equations)	
14	Distance Formula, Midpoint and Circles Perpendicular Bisector	Chapter 9, Section 9.1, pages 746-751 3 , completing the square and shifting, 6(appropriate guiding points)	p. 751: 5,9,11,13,23-31 odd, 39,41,45,59,61,63,67,71 Supplemental Problems on Perpendicular Bisector
15	Systems of three Linear Equations in Three Variables	Chapter 3, Section 3.6, pages 278-285.	p. 286: 11-17 odd, 21, 23, 27, 33-37 odd
16	Determinants and Cramer's Rule (optional) Systems involving Nonlinear Equations	Appendix A.1, pages A-1 to A-9. Chapter 9, Section 9.4, pages 776-780.	p. A-10: 35-45 odd, 49, 55, 57. p. 782: 23-37 odd, 49
17	**Midterm Examination**		1 session
18	Angle Measure and Special Triangles The Trigonometry of Right Triangles	Chapter 1, Section 1.1, pages 2-6 Chapter 2, Section 2.1, pages 46-50	p. 7: 45-57 odd p. 51: 7-21 odd
19	Solving Right Triangles Applications of Static Trigonometry	Chapter 2, Section 2.2, pages 54-56 Chapter 2, Section 2.3, pages 63-66	p. 57: 7-47 odd p. 69: 35-38 all
20	Angle Measure in Radian Trigonometry and the Coordinate Plane	Chapter 3, Section 3.1, pages 90-93 Chapter 1, Section 1.3, pages 22-27	p. 95: 25-39 odd, 43, 45, 49-61 odd, 67-71 odd p. 28: 25-31 odd, 45, 47, 55-63 odd, 64, 73-79 odd
21	Unit Circles	Chapter 3, Section 3.3, pages 108-113	p. 115: 29-35 odd, 37-40 all
22	Graphs of the Sine and Cosine Functions Graphs of Tangent and Cotangent Functions (optional)	3, 6(key features) Chapter 4, Section 4.1, pages 134-144 Chapter 4, Section 4.2, pages 153-159	p. 145: 1 – 3 all, 17-29 odd, 33-39 odd p. 160: 15,19,21,39,43,47
23	Fundamental Identities and Families of Identities	Chapter 1, Section 1.4, pages 31-35 Chapter 5, Section 5.1, pages 212-214	p. 35: 11-37 odd p. 216: 13-29 odd,37,43,51
24	Trigonometric Equations	Chapter 6, Section 6.3, pages 284-290	p. 292: 13,17,21,25,31,35,43-49 odd, 79, 80
25	Oblique Triangles and the Law of Sines The Law of Cosines	Chapter 7, Section 7.1, pages 316-322 Chapter 7, Section 7.2, pages 329-332	p. 324: 7-23 odd p. 338: 7-11 odd, 21-29 odd
26	**Third Examination** Exponential Functions	Chapter 8, Subsections 8.3.1, 8.3.2, 8.3.4.	p. 678: 9-25 odd, 43, 49
27	Logarithmic Functions	Chapter 8, Section 8.4, pages 682-685 and example 8, 9.	p. 690: 11-61 odd
28	Properties of Logarithms Compound Interest	Chapter 8, Section 8.5, pages 696-700. Chapter 8, Section 8.6, pages 704-707 (omit example 3).	p. 701: 17-29 odd, 45-55 odd, 63-69, 77, 79, 89 p. 712: 11, 13

29	Exponential Equations	Chapter 8, Section 8.7, pages 721-726.	p. 726: 39-49 odd, 55- 61 odd, 73, 75, 77, 79, 85
30	**Final Examination**		1 session